

# CBCS SCHEME

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17CS36

## Third Semester B.E. Degree Examination, Feb./Mar. 2022 Discrete Mathematical Structures

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

### Module-1

- 1 a. Write the converse, inverse and contrapositive for the following implication. For each, determine the truth value: "If  $0 + 0 = 0$  then  $1 + 1 = 1$ " (06 Marks)
- b. State the rule of universal specification. Show that the following argument is valid.
- $$\begin{array}{l} \neg r(c) \\ \forall t([p(t) \rightarrow q(t)]) \\ \forall t([q(t) \rightarrow r(t)]) \\ \hline \therefore \neg p(c) \end{array}$$
- (06 Marks)
- c. Prove the following without using Truth table:
- i)  $(p \rightarrow q) \wedge [\neg q \wedge (r \vee \neg q)] \Leftrightarrow \neg(q \vee p)$
- ii)  $p \rightarrow (q \rightarrow q) \Leftrightarrow (p \wedge q) \rightarrow r$  (08 Marks)

OR

- 2 a. If a proposition  $q$  has the truth value 1, determine all truth value assignment for the primitive propositions  $p, r$  and  $s$  for which the truth value of the following compound proposition is 1.  
 $[q \rightarrow \{(\neg p \vee r) \wedge \neg s\}] \wedge \{\neg s \rightarrow (\neg r \wedge q)\}$  (06 Marks)
- b. Prove the statement "If  $n$  is an odd integer then  $n + 9$  is an even integer" using :  
(i) Direct method (ii) Indirect method (iii) Proof by contradiction (06 Marks)
- c. Show that the following arguments are valid using Rules of inferences: (08 Marks)
- i)  $\begin{array}{l} p \rightarrow q \\ r \rightarrow s \\ p \vee r \\ \hline \therefore q \vee s \end{array}$
- ii)  $\begin{array}{l} \neg p \Leftrightarrow q \\ q \rightarrow r \\ \hline \neg r \\ \hline \therefore p \end{array}$

### Module-2

- 3 a. Prove by mathematical induction  
 $1^2 + 3^2 + 5^2 + \dots + (2n-1)^2 = \frac{1}{3}n(2n-1)(2n+1)$  (06 Marks)
- b. Find the number of permutations of the letters of the word MASSASAVGA. In how many of these all four A's are together? How many of them begin with S? (06 Marks)
- c. Find the coefficient of : (08 Marks)
- (i)  $x^9 y^3$  in the expansion of  $(2x - 3y)^{12}$  (ii)  $x^0$  in the expansion of  $\left(3x^2 - \frac{1}{x}\right)^{15}$

OR

- 4 a. Prove that  $4n < (n^2 - 7)$  for all positive integers  $n \geq 6$ . (06 Marks)
- b. A box contains 15 IC chips of which 7 are defective and 8 are non-defective. In how many ways 5 chips can be chosen so that:  
(i) All are non defective (ii) All are defective (iii) 2 are non-defective (06 Marks)

- c. In how many ways can 10 identical pencils are distributed among 5 children in the following cases:
- These are no restriction
  - Each child get atleast one pencil
  - The youngest child gets atleast 2 pencils.
- (08 Marks)

**Module-3**

- 5 a. Define relation. Let A and B be finite sets with  $|A| = 3$ . If these are 4096 relations from A to B, what is  $|B|$ ? (06 Marks)
- b. Define equivalence relation. Let S be the set of all non zero integers and  $A = S \times S$  on A define the relation R by  $(a, b) R (c, d)$  iff  $ad = bc$ . Show that R is an equivalence relation. (06 Marks)
- c. Define partial order relation. Let  $A = \{1, 2, 3, 4, 6, 8, 12\}$  and R be the partial ordering on A defined by  $aRb$  iff a divides b.
- Construct Hasse diagram
  - Find maximal and minimal elements
  - Find upper bounds and lower bounds if the subset  $B = \{2, 3, 6\}$
- (08 Marks)

**OR**

- 6 a. Define one to one and onto functions. Let the set  $A = \{a, b, c, d, e, f, g\}$  and  $B = \{1, 2, 3, 4\}$ . How many onto functions exist from A to B? (06 Marks)
- b. Let  $A = \{1, 2, 3, 4\}$  and let R be the relation on A defined by  $xRy$  iff "x divides y".
- Write the relation R
  - Draw the digraph of R and matrix of R
- (06 Marks)
- c. State pigeonhole principle. Prove that if 101 integers are selected from the set  $S = \{1, 2, 3, \dots, 200\}$  then atleast two of these are such that one divides the other. (08 Marks)

**Module-4**

- 7 a. Determine the number of positive integers n such that  $1 \leq n \leq 100$  and n is not divisible by 2, 3 or 5. (06 Marks)
- b. Find the Rook polynomial for the  $3 \times 3$  board using expansion formula. (06 Marks)
- c. Solve the recurrence relation to find the  $n^{\text{th}}$  Fibonacci number. (08 Marks)

**OR**

- 8 a. Define Dérangements. Find the number of derangements of 1, 2, 3, 4. (06 Marks)
- b. How many integers between 1 and 300 (inclusive) are:
- Divisible by atleast one of 5, 6, 8?
  - Divisible by none of 5, 6, 8?
- (06 Marks)
- c. Four persons  $P_1, P_2, P_3$  and  $P_4$  who arrive late for a dinner party find that only one chair at each of 5 tables  $T_1, T_2, T_3, T_4$  and  $T_5$  is vacant.  $P_1$  will not sit at  $T_1$  or  $T_2$ ,  $P_2$  will not sit at  $T_2$ ,  $P_3$  will not sit at  $T_3$  or  $T_4$  and  $P_4$  will not sit at  $T_4$  or  $T_5$ . Find the number of ways they can occupy the vacant chairs. (08 Marks)

**Module-5**

- 9 a. Construct an optimal prefix code for the symbols a, o, q, u, y, z that occur with frequencies 20, 28, 4, 17, 12, 7 respectively. (08 Marks)
- b. Write notes on: (i) Spanning subgraph (ii) Induced subgraph (iii) Isomorphic graphs (12 Marks)

**OR**

- 10 a. Define complete graph. Show that a complete graph with n vertices has  $n(n-1)/2$  edges. (06 Marks)
- b. What is a regular graph? If a graph with n vertices and m edges is K-regular. Show that  $M = Kn/2$ . (06 Marks)
- c. Write notes on: (i) Euler circuits and Euler trails (ii) Connected and disconnected graph (08 Marks)